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Generation of Alfven Waves in the Magnetosphere by Parametric Interaction between Whistlers

by

K. J. Harker, F. W. Crawford and A. C. Fraser-Smith

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ABSTRACT

A theory is developed for the generation of Alfven waves by the mixing of whistler waves. Calculations are carried out for interaction in the magnetosphere where the whistlers fall in the VLF frequency range, and the Alfven wave in the ULF (1-5 Hz) range. Typical amplitudes of 2-5 mγ for the Alfven wave are calculated, and it is shown that these values might be increased by one order of magnitude through suitable variation of the experimental parameters. The study thereby develops a possible explanation for naturally occurring Pc 1 micropulsations, and demonstrates the feasibility of artificial generation of such micropulsations by ground-launched whistlers.

CONTENTS

FIGURES

1. Introduction.

A number of studies have been made of nonlinear three-wave interaction between circularly-polarized waves such as whistlers [Harker and Crawford, 1969, Kim et al, 1971]. In that case all three waves are high frequency, i.e. they depend primarily on the electron rather than the ion motions. Another related problem of equal importance is the possibility of interaction involving a low frequency wave, i.e. a mode in which ion motions play a role. In this paper we shall consider the interaction of two whistlers to produce an Alfven wave. We will restrict ourselves to the study of waves propagating at small angles, θ , to the geomagnetic field, in which case the interaction is proportional to θ . This problem has been previously studied by Suramlishvili [1967] for the limiting case where the angles, θ , are zero and the interaction vanishes.

Although our analysis will be quite general, our primary concern will be with the magnetosphere as the medium in which interaction takes place. In this case the whistlers lie in the VLF, and the Alfven wave in the ULF (1-5 Hz) frequency range. Our purpose in studying this region is two-fold. First, we will show that naturally occurring whistlers can generate Pc 1 micropulsations which fall within the observed intensity range of 1-1000 mγ [Jacobs, 1970], and thereby demonstrate the possibility of a mechanism for the generation of such naturally occurring micropulsations. Second, our calculations will demonstrate the feasibility of using whistlers launched from ground-based transmitters for the generation of artificial micropulsations.

The geometry to be considered is shown in Fig. 1. Two whistlers follow a trajectory along a geomagnetic field line. In a region of width L, located a distance Z along the magnetic field line from the equatorial plane, the difference frequency and wavenumber correspond to a propagating Alfven wave. Because of this resonant condition, the two whistlers mix to produce Alfven waves, which then propagate out of the region as shown in the figure.

As mentioned above, we will restrict ourselves to the study of waves propagating at small angles to the geomagnetic field. Under these circumstances we are concerned with the waves whose dispersion curves are plotted in Figure 2. The upper branch is a whistler, while the lower is an ion-cyclotron wave. In the region $0 < \omega \ll \omega_{ci}$, where ω_{ci} is the ion gyrofrequency, the curves coalesce to form the region corresponding to Alfven waves. The dispersion relation for the whistler is given by [Clemmow and Dougherty, 1969],

$$K^2 = \Omega^2 - \frac{\Omega}{(\Omega-\Omega_{ce})} \quad , \tag{1}$$

and the Alfven wave by

$$K^2 = \frac{\Omega^2}{\Omega_{ce}\Omega_{ci}} \quad . \tag{2}$$

In these equations we have used normalized units for the wavenumber, frequency, and the electron and ion gyrofrequencies,

$$K = kc/\omega_0 \qquad \Omega = \omega/\omega_0 \qquad \Omega_{ce} = \omega_{ce}/\omega_0 \qquad \Omega_{ci} = \omega_{ci}/\omega_0 \ , \tag{3}$$

where

$$\omega_{ce} = \frac{e|B_0|}{m_e} \ , \quad \omega_{ci} = \frac{e|B_0|}{m_i} \ , \quad \omega_0 = \frac{ne^2}{\overline{m}\,\epsilon_0} \ , \quad \overline{m} = \frac{m_e m_i}{m_e + m_i} \ . \tag{4}$$

As mentioned above, the two waves which mix will be whistlers, and the product wave of the mixing will be the Alfven wave. We will denote the Alfven wave by subscript 1, the lower frequency whistler by subscript 2, and the higher frequency whistler by subscript 3.

2. Synchronism Conditions.

For nonlinear interaction to occur, it is necessary that the synchronism conditions

$$K_3 = K_2 + K_1 \quad , \qquad \Omega_3 = \Omega_2 + \Omega_1 \quad , \tag{5}$$

be satisfied, i.e., the points (0, 0), (K_3, Ω_3), (K_2, Ω_2), (K_1, Ω_1) must form the vertices of a parallelogram in the (K, Ω) plane, as shown in Figure 2. Since the Alfven waves have a linear dispersion relation, it is clear from the figure that, for sufficiently small K_1 and Ω_1, (K_2, Ω_2) and (K_3, Ω_3) lie on that portion of the whistler dispersion curve where the group velocity equals the Alfven wave velocity. From Eqs. (1) and (2), K_2, K_3, Ω_2, Ω_3 are given by

$$K_2 \cong K_3 \cong K_0 \; , \qquad \Omega_2 \cong \Omega_3 = \Omega_0 \quad , \tag{6}$$

where K and Ω are determined by the simultaneous solution of the equations

$$K_0^{\;2} = \Omega_0^{\;2} + \frac{\Omega_0}{\Omega_{ce} - \Omega_0} \; , \tag{7}$$

$$2\, K_0 \frac{dK_0}{d\Omega_0} = 2\, \Omega_0 + \frac{\Omega_{ce}}{(\Omega_{ce} - \Omega_0)^2} \; , \tag{8}$$

$$\frac{dK_0}{d\Omega_0} = \frac{1}{(\Omega_{ce}\Omega_{ci})^{1/2}} \; . \tag{9}$$

Since $(\Omega_{ce}-\Omega_0) \ll 1$, we may neglect the term Ω_0^2 in Eq (7) and obtain

$$K_0^2 + 1 = \frac{\Omega_{ce}}{\Omega_{ce}-\Omega_0} . \tag{10}$$

For the same reason we may neglect the term 2Ω in Eq.(8) and obtain

$$2\left(\frac{\Omega_{ce}}{\Omega_{ci}}\right)^{1/2} K_0 = \left(\frac{\Omega_{ce}}{\Omega_{ce}-\Omega_0}\right)^2 . \tag{11}$$

Combining these gives the final equation for K_0: (12)

$$(K_0^2 + 1)^2 = 2\left(\frac{\Omega_{ce}}{\Omega_{ci}}\right)^{1/2} K_0 .$$

From Eq. (10) we obtain the corresponding value for Ω_0 in the form

$$\frac{\Omega_0}{\Omega_{ce}} = \frac{K_0^2}{1+K_0^2} . \tag{13}$$

3. <u>Coupled Mode Equation.</u>

The growth of the Alfven wave in the interaction region is governed by a well-known coupled mode equation which we may write in the notation of <u>Galloway and Crawford</u> (1970) as

$$\frac{i\Gamma_1}{\Omega_1}\left(\underset{\sim}{V}_{g1} \cdot \frac{\partial E_{p1}}{\partial \underset{\sim}{R}}\right) = \Gamma_0^* E_{p2}^* E_{p3} , \tag{14}$$

in terms of the dimensionless variables

$$\underset{\sim}{V}_{g1} = \underset{\sim}{v}_{g1}/c \ , \quad \underset{\sim}{R} = \underset{\sim}{r}\omega_0/c \ , \quad E_p = \hat{E}_p \ e/\bar{m} \ \omega_0 c \tag{15}$$

where $\underset{\sim}{v}_{g1}$ is the group velocity, $\underset{\sim}{r}$ is distance, and $\hat{E}_{p1,2,3}$ is the component of the wave electric field perpendicular to the plane containing both $\underset{\sim}{K}$ and $\underset{\sim}{B}$. Constants $\Gamma_1 \underset{\sim}{V}_{g1}$ and Γ_0^* are defined by Galloway and Crawford [1970], and will be determined in the next section.

We next set up a rectangular coordinate system for $\underset{\sim}{R}$ with Z along the geomagnetic field. Because all waves are assumed to propagate at small angles to the geomagnetic field, we may ignore the traverse components of the product on the left-hand side of Eq. (14), and then integrate along Z to obtain

$$E_{p1} = \frac{iL\Omega_1}{\Gamma_1 \underset{\sim}{V}_{g1}} \Gamma_0^* \ E_{p2}^* \ E_{p3} \ , \tag{16}$$

where $L = \int dZ$ is the width of the interaction region. In a recent paper [Harker and Crawford, 1970] it has been shown that

$$L = (2\pi/\beta)^{1/2} \qquad \beta = \left(\frac{c}{\omega_0}\right)^2 \left.\frac{\partial(k_3-k_2-k_1)}{\partial z}\right|_r \ . \tag{17}$$

Here subscript r denotes that the derivative is evaluated at the point in space where the synchronism conditions are exactly satisfied and $z = Zc/\omega_0$. Non-zero values of β arise from inhomogeneities in B_0 and the plasma density. This equation is based on the supposition that $(k_3-k_2-k_1)$ can be approximated by a linear function of z.

We may easily evaluate β by using the relation

$$\left.\frac{d(k_3-k_2-k_1)}{dz}\right|_r = \left\{\frac{\partial(k_3-k_2-k_1)}{\partial\omega_{ce}}\frac{d\omega_{ce}}{dz} + \frac{\partial(k_3-k_2-k_1)}{\partial\omega_o}\frac{d\omega_0}{dz}\right\}_r , \tag{18}$$

the dispersion relations,

$$k_{2,3} = \frac{\omega_0}{c}\left(\frac{\omega_{2,3}}{\omega_{ce}-\omega_{2,3}}\right)^{1/2}, \qquad k_1 = \frac{\omega_0}{c}\frac{\omega_1}{(\omega_{ce}\omega_{ci})^{1/2}} , \tag{19}$$

and the subsequent synchronism condition

$$k_3-k_2-k_1 = \frac{\omega_0}{c}\left[\left(\frac{\omega_3}{\omega_{ce}-\omega_3}\right)^{1/2} - \left(\frac{\omega_2}{\omega_{ce}-\omega_2}\right)^{1/2} - \frac{\omega_1}{(\omega_{ce}\omega_{ci})^{1/2}}\right] . \tag{20}$$

Let us concentrate on the first term in Eq. (18), which arises from inhomogeneity in the geomagnetic field. Differentiating Eq. (20) with respect to ω_{ce}, and substituting Eqs. (13) and (19) gives

$$\left.\frac{d(k_3-k_2-k_1)}{d\omega_{ce}}\right|_r = -\frac{1}{2c\Omega_{ce}}\left\{\frac{K_3^{\,2}+1}{K_3^{\,2}}K_3^{\,3/2} - \frac{K_2^{\,2}+1}{K_2^{\,2}}K_2^{\,3/2} - 2K_1\right\}_r . \tag{21}$$

Using the relation $K_3 = K_2 + K_1$, and remembering that $K_1 \ll K_2$, we can write the first term on the RHS of this equation as

$$\left(\frac{K_3^{\,2}+1}{K_3^{\,2}}\right)K_3^{\,3/2} = \left(\frac{K_2^{\,2}+1}{K_2^{\,2}}\right)K_2^{\,3/2} + \left[3K_2^{1/2} - K_2^{\,-3/2}\right]\frac{K_1}{2} . \tag{22}$$

This reduces Eq. (21) to the form

$$\left.\frac{d(k_3-k_2-k_1)}{d\omega_{ce}}\right|_r = \frac{K_1}{4c\Omega_{ce}}\left[4-3K_0^{1/2} + K_0^{-3/2}\right] , \tag{23}$$

if we replace K_2 by its approximate value, K . The factor $d\Omega_{ce}/dz$ is evaluated from the formula for the geomagnetic field given by Helliwell [1967] ,

$$d\omega_{ce} = \left(\frac{9\omega_{c0}}{r_m^2}\right) z\ dz , \tag{24}$$

where ω_{c0} is the electron gyrofrequency on the equator, and r_m is the distance from earth's center. If we approximate by replacing ω_{c0} by ω_{ce} on the RHS of this equation, we obtain

$$\frac{d\omega_{ce}}{dz} = \frac{9\omega_0^2}{c}\ \frac{Z\Omega_{ce}}{R_m^2} . \tag{25}$$

We now show that the second term on the RHS of Eq. (18) is zero. Differentiating Eq. (20) with respect to ω and substituting Eq. (19) gives

$$\left.\frac{\partial(k_3-k_2-k_1)}{\partial\omega_0}\right|_r = \frac{1}{c}\ (K_3-K_2-K_1)_r = 0 . \tag{26}$$

The final formula for the interaction length is then simply obtained by substituting Eqs. (23) and (25) into Eqs. (17) and (18), yielding

$$\beta = \frac{K_1 Z}{R_m^2}\ f(K_0) , \tag{27}$$

where

$$f(K_0) = \frac{9}{4}\ (4 - 3K_0^{1/2} + K_0^{-3/2}) . \tag{28}$$

The case of interaction on the equator requires special treatment, since the function $(k_3-k_2-k_1)$ is no longer a linear function of z, but quadratic instead. It is shown in Appendix A that the interaction length in this case is given by

$$L = \frac{(6/\gamma)^{1/3}\,\Gamma(1/3)}{3^{1/2}} , \tag{29}$$

where

$$\gamma = \left.\frac{\partial^2(K_3-K_2-K_1)}{\partial Z^2}\right|_r = \left(\frac{c}{\omega_o}\right)^3 \left.\frac{\partial^2(k_3-k_2-k_1)}{\partial Z^2}\right|_r , \tag{30}$$

and $\Gamma(1/3) = 2.68$ is the gamma function [Abramowitz and Stegun, 1964].

Straightforward differentiation shows that

$$\left.\frac{d^2(k_3-k_2-k_1)}{dz^2}\right|_r = \left\{ \frac{\partial^2(k_3-k_2-k_1)}{\partial\omega_{ce}^2}\left(\frac{d\omega_{ce}}{dz}\right)^2 + 2\,\frac{\partial^2(k_3-k_2-k_1)}{\partial\omega_{ce}\partial\omega_0}\,\frac{d\omega_0}{dz}\,\frac{d\omega_{ce}}{dz} \right.$$

$$\left. + \frac{\partial^2(k_3-k_2-k_1)}{\partial\omega_0^2}\left(\frac{d\omega_0}{dz}\right)^2 + \frac{\partial(k_3-k_2-k_1)}{\partial\omega_{ce}}\,\frac{d^2\omega_{ce}}{dz^2} + \frac{\partial(k_3-k_2-k_1)}{\partial\omega_0}\,\frac{d^2\omega_0}{dz^2} \right\}_r . \tag{31}$$

Because of the symmetry of the magnetic field about the equator it is cle: that

$$\left.\frac{d\omega_{ce}}{dz}\right|_r = 0 . \tag{32}$$

A simple extension of the argument leading to Eq. (26) shows furthermore that

$$\left.\frac{\partial^2(k_3-k_2-k_1)}{\partial\omega_0^2}\right|_r = 0 . \tag{33}$$

Substituting Eqs. (26), (32), and (33) reduces Eq. (31) to

$$\left.\frac{d^2(k_3-k_2-k_1)}{dz^2}\right|_r = \left\{\frac{\partial(k_3-k_2-k_1)}{\partial\omega_{ce}} \frac{d^2\omega_{ce}}{dz^2}\right\}_r \quad , \tag{34}$$

while from Eq. (24) we can show that

$$\left.\frac{d^2\omega_{ce}}{dz^2}\right| = \frac{9\omega_0^3}{c^2} \frac{\Omega_{ce}}{R_m^2} \ . \tag{35}$$

Combining Eqs. (23), (28), (30), (34), and (35) yields for γ the equation

$$\gamma = \frac{K_1}{R_m^2} f(K_0) \ . \tag{36}$$

4. <u>Constants of the Coupled Mode Equation.</u>

In this section we calculate the constants $\Gamma_1 \underset{\sim}{V}_{g1}$ and Γ_0 that appear in Eq. (16). It is convenient to follow the same line of attack as <u>Galloway and Crawford</u> [1970], with inclusion of ion motion. We write the scalar and vector potentials as

$$V = \hat{V}e/\bar{m}c^2 \ , \qquad A = \hat{\underset{\sim}{A}}e/\bar{m}c \quad , \tag{37}$$

the particle displacement vector as

$$\underset{\sim}{\Lambda}_e = \delta\underset{\sim}{r}_e \ \omega_0/c \ , \quad \underset{\sim}{\Lambda}_i = \delta\underset{\sim}{r}_i \ \omega_0/c \ , \tag{38}$$

the equilibrium position and the time as

$$\underset{\sim}{R}_0 = \underset{\sim}{r}_0 \ \omega_0/c \ , \qquad T = \omega_0 \ t \ , \tag{39}$$

where $\hat{V}$, $\hat{A}$, $\delta\underset{\sim}{r}$, $\underset{\sim}{r}_0$, and t are the corresponding dimensional physical quantities.

In terms of these variables, we may write the Lagrangian density as

$$\mathcal{L} = \frac{1}{2}\frac{m_e}{\bar{m}}\dot{\Lambda}_e^2 + \frac{1}{2}\frac{m_i}{\bar{m}}\dot{\Lambda}_i^2 + [V - \dot{\Lambda}_e \cdot A + \dot{\Lambda}_i \cdot A] + \frac{1}{2}[\nabla V + \dot{A}]^2 - \frac{1}{2}[\nabla \times A]^2 . \tag{40}$$

Both V and A are functions of $R_0 + \Lambda_{e,i}$, while all other terms are functions of R_0 only. Expansion of Eq. (40) as $\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_1 + \mathcal{L}_2 + \cdots$ yields

$$\mathcal{L}_2 = \frac{1}{2}\frac{m_e}{\bar{m}}\dot{\Lambda}_e^2 + \frac{1}{2}\frac{m_i}{\bar{m}}\Lambda_i^2 - (\dot{\Lambda}_e - \dot{\Lambda}_i)\cdot A_1 + \frac{1}{2}\dot{A}_1^2 - \frac{1}{2}(\nabla \times A_1)^2 , \tag{41}$$

$$\mathcal{L}_3 = -\dot{\Lambda}_e \cdot \left(\Lambda_e \cdot \frac{\partial}{\partial R_0}\right) A_1 + \dot{\Lambda}_i \cdot \left(\Lambda_i \cdot \frac{\partial}{\partial R_0}\right) A_1 , \tag{42}$$

where A has been replaced by a background, A_0, and a perturbation A_1 and a gauge has been chosen so that V is a constant $\left(\nabla\cdot A = -\int_0^T (N_e+N_i)dT'\right)$. Since the second term in Eq. (42) can be shown to be smaller than the first by the ratio $(m_e/m_i)^2$, it can be ignored, and $\mathcal{L}_3$ can be written simply as

$$\mathcal{L}_3 = -\dot{\Lambda}_e \cdot \left(\Lambda_e \cdot \frac{\partial}{\partial R_0}\right) A_1 . \tag{43}$$

For small-signal propagation as $\exp i(\Omega T - K \cdot R_0)$ we have [Clemmow and Dougherty, 1969]

$$\begin{bmatrix} S - \Pi^2 \cos^2\theta & iD & \Pi^2 \cos\theta \sin\theta \\ -iD & S - \Pi^2 & 0 \\ \Pi^2 \cos\theta \sin\theta & 0 & P - \Pi^2 \sin^2\theta \end{bmatrix} \cdot E = 0 , \tag{44}$$

where

$$\Pi^2 = K^2/\Omega^2 , \qquad P = 1 - 1/\Omega^2 , \tag{45}$$

$$S = 1 - \frac{\Omega^2 - \Omega_{ce}\Omega_{ci}}{(\Omega^2 - \Omega_{ce}^2)(\Omega^2 - \Omega_{ci}^2)} \qquad D = -\frac{\Omega(\Omega_{ce} - \Omega_{ci})}{(\Omega^2 - \Omega_{ce}^2)(\Omega^2 - \Omega_{ci}^2)},$$

and K lies in the $X - Z$ plane at angle θ to the Z-axis. A rotation operator, R, will be used to generalize the propagation to spherical angles θ, ϕ as shown in Fig. 3. For $0 < \theta < \pi/2$, all $\underset{\sim}{E}$ components are nonzero. We will choose to express $\underset{\sim}{\Lambda}_{e,i}$ and $\underset{\sim}{A}_1$ in terms of the component E_p perpendicular to $\underset{\sim}{K}$ and the Z-axis. We then have

$$\underset{\sim}{\Lambda} = \underset{\approx}{\Sigma}\, E_p, \quad \underset{\sim}{A}_1 = \underset{\sim}{U}\, E_p, \quad \underset{\approx}{\Sigma} = R\,\underset{\sim}{\Sigma}', \quad \underset{\approx}{U} = R\underset{\sim}{U}', \tag{46}$$

where

$$R = \begin{vmatrix} \cos\phi & -\sin\phi & 0 \\ \sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{vmatrix}, \tag{47}$$

$$\sum{}' = \frac{\bar{m}}{m} \begin{vmatrix} \dfrac{-j}{\Omega D(\Omega^2 - \Omega_{ce}^2)} \{-\Omega_{ce} D + \Omega(S - \Pi^2)\} \\[2ex] \dfrac{-1}{\Omega D(\Omega^2 - \Omega_{ce}^2)} \{-\Omega_{ce}(\Pi^2 - S) - \Omega D\} \\[2ex] \dfrac{j}{\Omega^2} \dfrac{\Pi^2 (S - \Pi^2) \cos\theta \sin\theta}{D(P - \Pi^2 \sin^2\theta)} \end{vmatrix}, \tag{48}$$

$$U' = \begin{vmatrix} \dfrac{S - \Pi^2}{\Omega D} \\[2ex] i/\Omega \\[2ex] -\dfrac{\Pi^2 (S - \Pi^2)\cos\theta\sin\theta}{\Omega D\,(P - \Pi^2 \sin^2\theta)} \end{vmatrix}. \tag{49}$$

The constants in the coupled mode equations are determined by the relations [Galloway and Crawford, 1970]

$$\frac{i}{8}\,\overline{\mathcal{L}_3(\Lambda_{1e}+\Lambda_{2e}+\Lambda^*_{3e},\ \Lambda_{1i}+\Lambda_{2i}+\Lambda^*_{3i},\ A_1+A_2+A^*_3)} + \text{c.c.} = i\Gamma_0 E_{p1}E_{p2}E^*_{p3}\,, \tag{50}$$

$$\underset{\sim}{\not{P}}_n = -\,\overline{\dot{\underset{\sim}{A}}_n \times \nabla \times \underset{\sim}{A}_n} = \underset{\sim}{V}_{gn}\,\Gamma_n\,\hat{E}_{pn}\,\hat{E}^*_{pn} \qquad (n=1,2,3)\,. \tag{51}$$

Substituting Eqs. (46) into Eqs. (43) and (51) then yields expressions for these constants in the form

$$\underset{\sim}{v}_{g1}\,\Gamma_1 = \Omega_1^2 \begin{Bmatrix} U_1^2\,\underset{\sim}{\Pi}_1/2 \\ -\,(\underset{\sim}{\Pi}_1\cdot\underset{\sim}{U}_1)\underset{\sim}{U}^*_1/4 \\ -\,(\underset{\sim}{\Pi}_1\cdot U^*_1)\underset{\sim}{U}_1/4 \end{Bmatrix}\,, \tag{52}$$

$$\Gamma_0 = -\,\frac{1}{8}\begin{Bmatrix} \Omega_1\,\underset{\sim}{\Sigma}_1\cdot[(\underset{\sim}{\Sigma}_2\cdot\underset{\sim}{K}_3)\underset{\sim}{U}^*_3 - (\underset{\sim}{\Sigma}^*_3\cdot\underset{\sim}{K}_2)\underset{\sim}{U}_2] \\ +\,\Omega_2\,\underset{\sim}{\Sigma}_2\cdot[(\underset{\sim}{\Sigma}_1\cdot\underset{\sim}{K}_3)\underset{\sim}{U}^*_3 - (\underset{\sim}{\Sigma}^*_3\cdot\underset{\sim}{K}_1)\underset{\sim}{U}_1] \\ +\,\Omega_3\,\underset{\sim}{\Sigma}^*_3\cdot[(\underset{\sim}{\Sigma}_1\cdot\underset{\sim}{K}_2)U_2 + (\underset{\sim}{\Sigma}_2\cdot\underset{\sim}{K}_1)\underset{\sim}{U}_1] \end{Bmatrix}\,. \tag{53}$$

In order to obtain the salient features of the interaction without becoming lost in a tangle of algebraic complexities, we make the specification that the wave vectors of all three waves lie in a plane which contains the magnetic field, $\underset{\sim}{B}_0$ as shown in Fig. 4. This means that we can take $\phi = 0$ and $R = 1$ for all three waves in Eqs. (46) and (47).

We next emphasize the implications of our assumption that the angles $\theta \ll 1$: since we are dealing here with transverse waves, this means that that portion of the matrix in Eq. (44) multiplying the transverse components of $\underset{\sim}{E}$ must be approximately zero, or equivalently, that

$$S - \Pi^2 = \pm D \ . \tag{54}$$

Substituting into Eqs. (48) and (49) then yields

$$\underset{\sim}{U} = \underset{\sim}{U}_{\perp} + \theta \underset{\sim}{U}_z \quad \underset{\sim}{\Sigma} = \underset{\sim}{\Sigma}_{\perp} + \theta \underset{\sim}{\Sigma}_z \quad , \tag{55}$$

where

$$\underset{\sim}{U} = \pm \frac{\underset{\sim}{a}^{\pm}}{\Omega} \ , \qquad \underset{\sim}{U}_z = \mp \frac{\Pi^2}{\Omega P} \underset{\sim}{i}_z \ ,$$

$$\underset{\sim}{\Sigma}_{\perp} = \mp \frac{\overline{m}}{m_e} \frac{j\underset{\sim}{a}^{\pm}}{\Omega(\Omega \pm \Omega_{ce})} \qquad \underset{\sim}{\Sigma}_z = \pm j \frac{\overline{m}}{m_e} \frac{\Pi^2}{\Omega^2 P} \underset{\sim}{i}_z \ , \tag{56}$$

$$\underset{\sim}{a}^{\pm} = \underset{\sim}{i}_x \pm j \underset{\sim}{i}_y$$

and $\underset{\sim}{i}_x$, $\underset{\sim}{i}_y$, and $\underset{\sim}{i}_z$ are unit vectors along the X, Y, and Z axes, respectively. The assumption that $\theta \ll 1$ also implies that

$$\underset{\sim}{K} = \theta \underset{\sim}{K}_{\perp} + \underset{\sim}{K}_z \quad , \tag{57}$$

where

$$\underset{\sim}{K}_{\perp} = K \underset{\sim}{i}_x \ , \quad \underset{\sim}{K}_z = K \underset{\sim}{i}_z \quad , \tag{58}$$

In particular, for whistlers we must take the lower sign in these equations, and the above vectors become

$$\underset{\sim}{U}_{\perp n} = - \frac{\underset{\sim}{a}^-}{\Omega_n} \qquad \underset{\sim}{U}_{zn} = - \frac{K_n^2}{\Omega_n} \underset{\sim}{i}_z \quad , \tag{59}$$

$$\underset{\sim}{\Sigma}_{\perp n} = \frac{\overline{m}}{m_e} \frac{j\underset{\sim}{a}^-}{\Omega_n(\Omega_n - \Omega_{ce})} \ , \ \underset{\sim}{\Sigma}_{zn} = j \frac{\overline{m}}{m_e} \frac{K_n^2}{\Omega_n^2} \underset{\sim}{i}_z \ , \ (n = 2,3) \ .$$

provided we assume

$$P_n = 1 - \frac{1}{\Omega_n^2} \simeq - \frac{1}{\Omega_n^2} \ . \tag{60}$$

For the Alfven wave we use Eqs. (2) and (60), and the condition

$$\Omega_1 << \Omega_{ce} \text{ and } \Omega_{ci} \ , \tag{61}$$

and the corresponding vectors become

$$\underset{\sim}{U}_{\perp 1} = \pm \underset{\sim}{a}^{\pm}/\Omega_1 \ , \qquad \underset{\sim}{U}_{z1} = \pm \frac{K_1}{(\Omega_{ce}\Omega_{ci})^{1/2}} \underset{\sim}{i}_z \ , \tag{62}$$

$$\underset{\sim}{\Sigma}_{\perp 1} = - \frac{\bar{m}}{m_e} \frac{j\underset{\sim}{a}^{\pm}}{\Omega_1 \Omega_{ce}} \ , \qquad \underset{\sim}{\Sigma}_{z1} = \mp j \frac{\bar{m}}{m_e} \frac{\underset{\sim}{i}_z}{\Omega_{ce}\,\Omega_{ci}} \ . \tag{63}$$

In order to evaluate Γ_0, we first expand it to lowest order in θ_1, θ_2 and θ_3. Substituting Eqs. (55) and (57) into Eq. (53), and retaining only terms up to first order in these angles, yields

$$\begin{aligned} \Gamma_0 = - \frac{1}{8} \Big\{ & \Omega_1 (\underset{\sim}{\Sigma}_{\perp 1} \cdot \underset{\sim}{U}^*_{\perp 3})(\theta_3 \underset{\sim}{\Sigma}_{\perp_2} \cdot \underset{\sim}{K}_{\perp 3} + \theta_2 \underset{\sim}{\Sigma}_{z2} \cdot \underset{\sim}{K}_{z3}) \\ & - \Omega_1 (\underset{\sim}{\Sigma}_{\perp 1} \cdot \underset{\sim}{U}_{\perp 2})(\theta_2 \underset{\sim}{\Sigma}^*_{\perp 3} \cdot \underset{\sim}{K}_{\perp 2} + \theta_3 \underset{\sim}{\Sigma}^*_{z3a} \cdot \underset{\sim}{K}_{z2}) \\ & + \Omega_2 (\underset{\sim}{\Sigma}_{\perp 2} \cdot \underset{\sim}{U}^*_{\perp 3})(\theta_3 \underset{\sim}{\Sigma}_{\perp 1} \cdot \underset{\sim}{K}_{\perp 3} + \theta_1 \underset{\sim}{\Sigma}_{zi} \underset{\sim}{K}_{z3}) \\ & - \Omega_2 (\underset{\sim}{\Sigma}_{\perp 2} \cdot \underset{\sim}{U}_{\perp 1})(\theta_1 \underset{\sim}{\Sigma}^*_{\perp 3} \cdot \underset{\sim}{K}_{\perp 1} + \theta_3 \underset{\sim}{\Sigma}^*_{z3} \cdot \underset{\sim}{K}_{z1}) \\ & + \Omega_3 (\underset{\sim}{\Sigma}^*_{\perp 3} \cdot \underset{\sim}{U}_{\perp 2})(\theta_2 \underset{\sim}{\Sigma}_{\perp 1} \cdot \underset{\sim}{K}_{\perp 2} + \theta_1 \underset{\sim}{\Sigma}_{z1} \cdot \underset{\sim}{K}_{z2}) \\ & + \Omega_3 (\underset{\sim}{\Sigma}^*_{\perp 3} \cdot \underset{\sim}{U}_{\perp 1})(\theta_1 \underset{\sim}{\Sigma}_{\perp 2} \cdot \underset{\sim}{K}_{\perp 1} + \theta_2 \underset{\sim}{\Sigma}_{z2} \cdot \underset{\sim}{K}_{z1}) \Big\} \ . \end{aligned} \tag{64}$$

We next expand Eq. (64) in powers of Ω_1 and K_1. Both Ω_1 and K_1 are of the same order because of the dispersion relation, Eq. (2). In carrying out this expansion we must eliminate all terms corresponding to Wave 2 in terms of Wave 3, or *vice versa*, since the differences involve terms of differing order in Ω_1 . We shall choose the former alternative. Starting with the synchronism conditions, given by

$$\Omega_2 = \Omega_3 - \Omega_1 , \qquad K_2 = K_3 - K_1 , \quad K_2 \theta_2 = K_3 \theta_3 - K_1 \theta_1, \tag{65}$$

we are thus led to the expansions

$$\theta_2 = \theta_3 + \frac{K_1}{K_3} (\theta_3 - \theta_1) + \cdots , \qquad \underset{\sim}{U}_{\perp 2} = \underset{\sim}{U}_{\perp 3} \left(1 + \frac{\Omega_1}{\Omega_3} + \ldots\right) , \tag{66}$$

$$\underset{\sim}{\Sigma}_{\perp 2} = \underset{\sim}{\Sigma}_{\perp 3} \left(1 + \frac{\Omega_1}{\Omega_3} + \frac{\Omega_1}{\Omega_3 - \Omega_{ce}} + \ldots\right) , \tag{67}$$

$$\underset{\sim}{\Sigma}_{z2} = \underset{\sim}{\Sigma}_{z3} \left(1 + \frac{2\Omega_1}{\Omega_3} - \frac{2K_1}{K_3} + \ldots\right) . \tag{68}$$

Substituting Eqs. (65)-(68) into Eq. (64), we observe that all terms are of order Ω_1^0 except the third and fifth, each of which is of order Ω_1^{-1} . However these terms of order Ω_1^{-1} cancel, and we are left to lowest order with terms of order Ω_1^0 . Collecting terms, simplifying, and replacing K_3 and Ω_3 by K_0 and Ω_0, respectively, finally leads to the expression

$$\Gamma_0 = - \frac{(\bar{m}/m_e)^2}{\Omega_{ce}^4} \left[\theta_3 \, g_3(K_0) - \theta_1 \, g_1(K_0)\right] \quad , \tag{69}$$

where

$$g_3(K_0) = - \frac{1}{4} \left\{ \frac{K_0}{\frac{\Omega_0}{\Omega_{ce}} \left(1 - \frac{\Omega_0}{\Omega_{ce}}\right)^2} + \frac{K_0^3}{(\Omega_0/\Omega_{ce})^3} - \frac{K_0^2 (\Omega_{ce}/\Omega_{ci})^{1/2}}{\left(\Omega_0/\Omega_{ce}\right)^2 \left(1 - \frac{\Omega_0}{\Omega_{ce}}\right)} \right\} ,$$

$$g_1(K_0) = \frac{(\Omega_{ce}/\Omega_{ci})^{1/2}}{4(1 - \Omega_0/\Omega_{ce})^2} \quad . \tag{70}$$

The remaining constant in the coupled mode equation, $\underset{\sim}{V}_{g1} \, \Gamma_1$, is readily obtained by substituting Eqs. (55) and (57) into Eq. (52). To lowest order in θ_1 we obtain

$$\underset{\sim}{V}_{g1} \, \Gamma_1 = \frac{1}{2} \, \Omega_1^2 \, U_{\perp 1}^{\,2} \, \Pi_{z1} \, \underset{\sim}{i}_z \quad , \tag{71}$$

or, after substitution of Eq. (62), simply

$$\underset{\sim}{V}_{g1} \, \Gamma_1 = (K_1/\Omega_1) \, \underset{\sim}{i}_z \quad . \tag{72}$$

5. Field Strength of the Alfven Wave

Substituting Eqs. (17), (27), (69), and (72) into Eq. (16) yields the final equation for the Alfven wave electric field strength in the form

$$E_{1p} = -\left[\frac{2\pi}{f(K_0)}\right]^{1/2} j \left(\frac{R_m^2\,\Omega_1}{Z}\right)^{1/2} \frac{(\bar{m}/m_e)^2[\theta_3 g_3(K_0) - \theta_1 g_1(K_0)]}{\Omega_{ce}^{5/2}\,(\Omega_{ce}/\Omega_{ci})^{3/4}}\, E_{2p}^{*}\, E_{3p} \;. \tag{73}$$

The associated wave magnetic field strength is determined by substituting Eq. (2) into Maxwell's equation

$$\underset{\sim}{K}_1 \times \underset{\sim}{E}_1 = \Omega_1\, \underset{\sim}{B}_1 \;. \tag{74}$$

This yields

$$|B_1| = \frac{(\Omega_{ce}/\Omega_{ci})^{1/2} E_{1p}}{\Omega_{ce}} \;. \tag{75}$$

It is useful to write these equations in dimensional form, in which case we obtain

$$\hat{E}_{1p} = -\left[\frac{2\pi}{f(K_0)}\right]^{1/2} j \left(\frac{r_m^2\,\omega_1}{zc}\right)^{1/2} \frac{(\bar{m}/m_e)^2\,\omega_0^{5/2}}{\omega_{ce}^{5/2}(\omega_{ce}/\omega_{ci})^{3/4}} \cdot \frac{[\theta_3 g_3(K_0) - \theta_1 g_1(K_0)]e}{\bar{m}c\,\omega_0}\, \hat{E}_{2p}^{*}\, \hat{E}_{3p} \,, \tag{76}$$

$$|\hat{B}_1| = \frac{(\omega_{ce}/\omega_{ci})^{1/2}\,\omega_0}{\omega_{ce}\, c}\, \hat{E}_{1p} \;. \tag{77}$$

One will notice from examining the above equations that the strength of the Alfven wave approaches infinity as the interaction region approaches the magnetic equator. This is merely a reflection of the fact that the assumption of a linear dependence of the sum $k_3-k_2-k_1$ on arc length embodied in Eq. (17) breaks down at the equator, where $k_3-k_2-k_1$ varies as the square of the arc length.

There is obviously some lower limit on how small z can become in the above equations. A suitable criterion is simply that the center of the interaction region can be no closer to the magnetic equator than the half width of the interaction region:

$$Z > L/2 \quad . \tag{78}$$

Substituting Eqs. (2), (17), and (27) into this inequality implies the criterion

$$\frac{z}{r_m} > \left[\frac{\pi\omega_{ce}\ c}{2f(K_0)\ \omega_1\omega_0\ r_m(\omega_{ce}/\omega_{ci})^{1/2}}\right]^{1/3} \quad . \tag{79}$$

When the interaction region is centered on the equator, we proceed as above but use Eqs. (29) and (36) in lieu of Eqs. (17) and (27). We then obtain the Alfven wave electric field in the form

$$E_{1p} = -\ j\ \frac{[6/f(K_0)]^{1/3}\ \Gamma(1/3)}{3^{1/2}} \left(R_m\ \Omega_1\right)^{2/3} \frac{(\bar{m}/m_e)^2[\theta_3 g_3(K_0)-\theta_1 g_1(K_0)]}{\Omega_{ce}^{8/3}\ \ (\Omega_{ce}/\Omega_{ci})^{2/3}}\ E_{2p}^*\ E_{3p} \ . \tag{80}$$

In dimensional form this becomes

$$\hat{E}_{1p} = -\ j\ \frac{[6/f(K_0)]^{1/3}\Gamma(1/3)}{3^{1/2}} \left(\frac{r_m\omega_1}{c}\right)^{2/3} \frac{(\bar{m}/m_e)^2\ \omega_0^{8/3}}{\omega_{ce}^{8/3}(\omega_{ce}/\omega_{ci})^{2/3}}\ \frac{[\theta_3 g_3(K_0)-\theta_1 g_1(K_0)]e}{\bar{m}c\ \omega_0}\ \hat{E}_{2p}^*\hat{E}_{3p} \ . \tag{81}$$

6. A Sample Calculation

In this section we shall use the above equations to estimate the strength of an Alfven wave generated by the mixing interaction. We will assume that the predominant ion is the proton so that

$$(\Omega_{ce}/\Omega_{ci})^{1/2} = 42.8. \tag{82}$$

Solving Eq. (12) for this value of the mass-ratio gives

$$K_0 = 4.25 \ . \tag{83}$$

From Eqs. (13), (28) and (70) we find that

$$\Omega_0/\Omega_{ce} = 0.948 \qquad f(K_0) = 4.67 \tag{84}$$

$$g_1(K_0) = 3910 \qquad g_3(K_0) = 3690 \ .$$

We will further assume that the field line along which the whistlers are propagating crosses the equator at four earth radii from the center of the earth so that

$$r_m = 2.545 \times 10^7 \text{ m} \ , \tag{85}$$

and that the interaction region is located at a distance from the equator given by

$$z/r_m = 1/9 \ . \tag{86}$$

Following Helliwell [1967] we take

$$\omega_0/2\pi = 180 \text{ kHz} \ , \qquad \omega_{ce}/2\pi = 14 \text{ kHz} \ , \tag{87}$$

for the plasma and electron gyrofrequencies. The electric field amplitude of the whistlers is taken to be

$$\hat{E}_{p2} = \hat{E}_{3p} = 1 \times 10^{-3} \text{ V/m} \ . \tag{88}$$

in accordance with the 'high field' case of Dysthe [1971].

The remaining parameters are assumed to be given by

$$\omega_1/2\pi \approx 1 \text{ Hz} \;,\; \theta_1 = 0 \text{ rad} \;,\; \theta_3 = 0.1 \text{ rad}. \tag{89}$$

Inserting these values into Eqs. (76) and (77) gives a value for the generated wave magnetic field of

$$|B| = 1.9 \text{ m}\gamma \;. \tag{90}$$

We must check that we have not chosen too small a value for z/r_m. Using the values assumed above, we find that Eq. (79) takes the form

$$\frac{z}{r_m} > 0.105 \;. \tag{91}$$

The value of 1/9 taken for z/r_m satisfies this inequality.

The same values [excepting, of course, Eq. (86)] when substituted into Eqs. (77) and (81) yield a value for the generated wave magnetic field of

$$|\hat{B}_1| = 2.4 \text{ m}\gamma \tag{92}$$

when the interaction region is centered on the equatorial plane.

7. Maximization of the Wave Fields

In Section 6 we have shown that an Alfven wave field of the order of 2 mγ can be generated for a typical set of magnetospheric parameters. Naturally occurring ULF waves fall in the range of 1-1000 mγ [Jacobs, 1970], so the question arises as to whether one can find conditions under which greater ULF wave intensities might be generated.

To investigate this question we shall maximize the field that can be generated with respect to z and ω_1. We assume that z is given by the minimal value permitted by Eq. (79) and use the approximation [Davies, 1969]

$$\omega_{ce} = \omega_{eq} \left(\frac{r_e}{r_m}\right)^3 \tag{93}$$

for the electron gyrofrequency, where $\omega_{eq}/2\pi$ is the gyrofrequency at the earth's surface on the equator (880 kHz) and r_e is earth's radius. Since our theory breaks down as ω_1 approaches ω_{ci}, we assume

$$\omega_1 = \omega_{ci}/2 \tag{94}$$

as the maximal value for the ULF frequency. Substituting into Eq. (76) and (77) yields the result

$$|\hat{B}_1| = \frac{[\pi/f(K_0)]^{1/3}}{c} \left(\frac{\omega_{ci}}{\omega_{ce}}\right)^{5/6} \left(\frac{r_m}{r_e}\right)^{5/3} \left(\frac{\omega_{eq} r_e}{c}\right)^{2/3} \left(\frac{\omega_0}{\omega_{ce}}\right)^{8/3} \left(\frac{\bar{m}}{m_e}\right)^2 \frac{\theta_3 g_3(K_0) - \theta_1 g_1(K_0)}{\omega_{eq} \bar{m}\ c/e}\ E^*_{2p} E_{3p} \ . \tag{95}$$

Using the same values for the variables in this formula as we used in Section 6, we arrive at a field strength of $|B_1| = 4.8$ mγ maximized with respect to ω_1 and z.

Let us now consider maximization in the case of the interaction region on the equatorial plane. Substituting Eqs. (93) and (94) into Eqs. (77) and (81) yields Eq. (95) again, except that the RHS must be increased by the factor $\Gamma(1/3)/[12^{1/6}\ \pi^{1/3}] \approx 1.2$. Thus we are led to the rather remarkable result that the two answers are identical in form with respect to their dependence on the experimental parameters, except for a numerical factor. Taking this fact into account, we arrive at a value of $|B_1| = 5.8$ mγ for interactions on the equatorial plane.

We now discuss how these values might be further maximized by variations of the remaining parameters. Since $\omega_0/\omega_{ce} \sim 10$ over most of the magnetosphere, little is to be gained here. The angle θ_3 might be increased to 0.2 radian, increasing the signal strength by a factor of two. Beyond 0.2 radian our approximations fail, and it is doubtful if further increase could be gained here. Since $|B_1| \sim (r_m/r_e)^{5/3}$, an increase, possibly as high as a factor of 3, could be made in the field strength by using more distant field lines. Overall, then, it might be possible to attain a field strength as high as 40 mγ.

Finally, no mention has been made of the fact that both whistlers and Alfven waves can be amplified during their transit along the field line. Values of the whistler field strength in excess of the 10^{-3} V/m used in Section 6 could occur at certain times and give correspondingly higher values of the generated ULF field. This ULF field could, in turn, also be amplified.

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Appendix

Here we will derive the interaction length for the special case where $K_3-K_2-K_1$ is a quadratic function of Z i.e.

$$K_3-K_2-K_1 = \frac{\gamma}{2} Z^2 \quad , \tag{A.1}$$

with γ as defined in Eq. (30). When synchronism is not exactly satisfied, Eq. (14) takes the more general form

$$\frac{i\Gamma_i}{\Omega_1}\left(\underset{\sim}{V}_{g1} \cdot \frac{\partial E_{p1}}{\partial \underset{\sim}{R}}\right) = \Gamma_0^* E_{p2}^* E_{p3} \exp\left(-j \int (\underset{\sim}{K}_3-\underset{\sim}{K}_2-\underset{\sim}{K}_1) \cdot \underset{\sim}{R}\, d\underset{\sim}{R}\right).$$

If we assume propagation nearly along Z, substitute Eq. (A.1), and integrate, we can reduce this equation to the form of Eq. (16) with

$$L = \int_{-\infty}^{\infty} \exp\left(- j\gamma Z^3/6\right) dZ \ . \tag{A.3}$$

Use of a standard integration formula [Gröbner and Hofreiter, 1961] reduces this expression for the integration length to Eq. (29).

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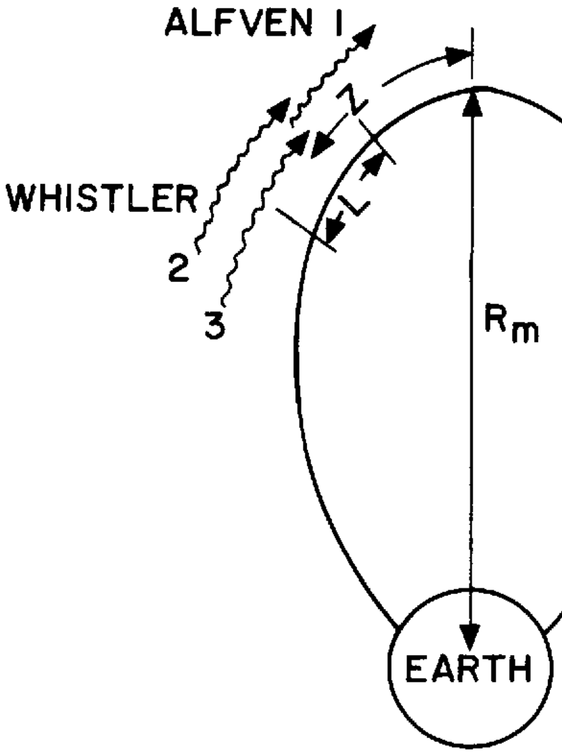


Figure 1

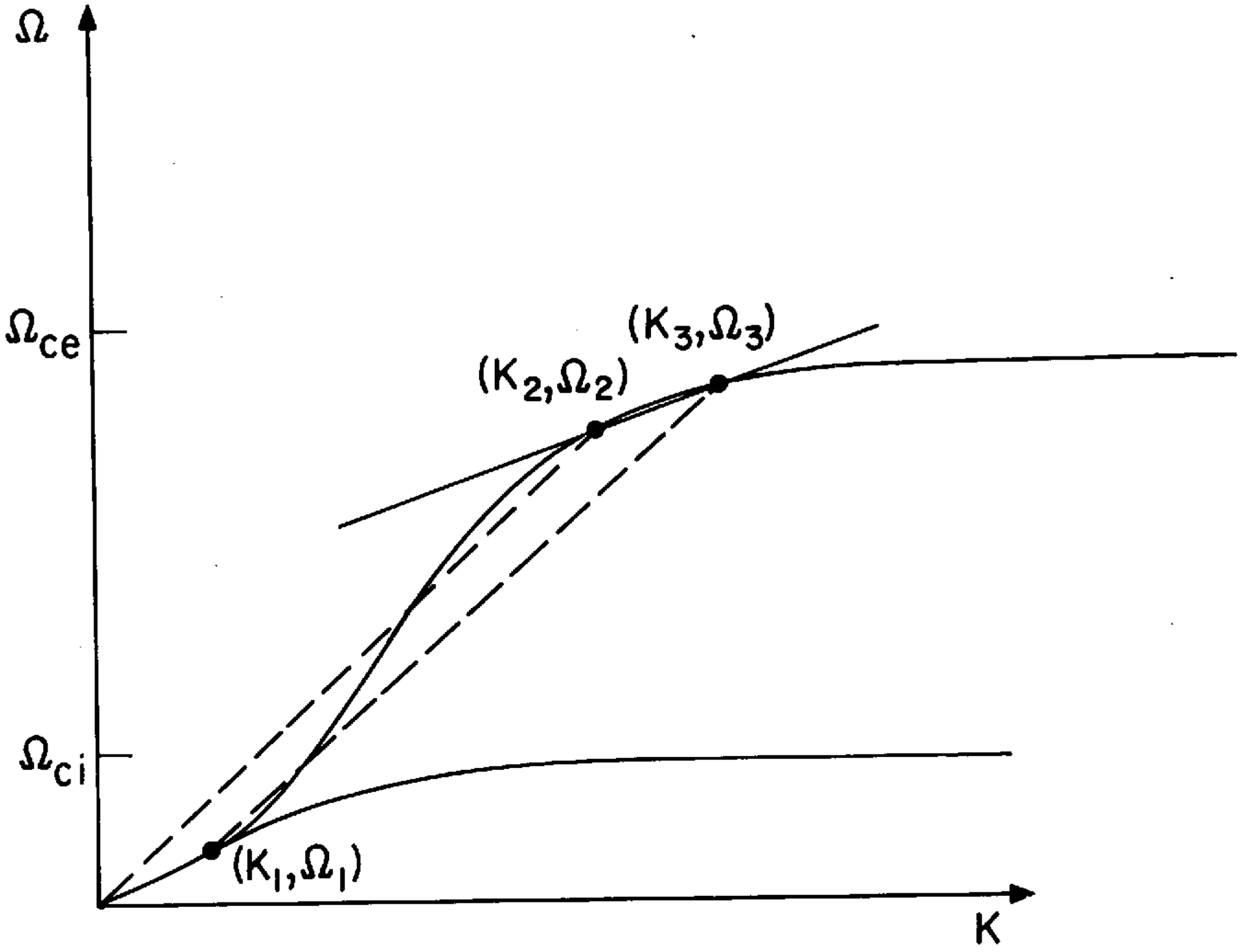


Figure 2

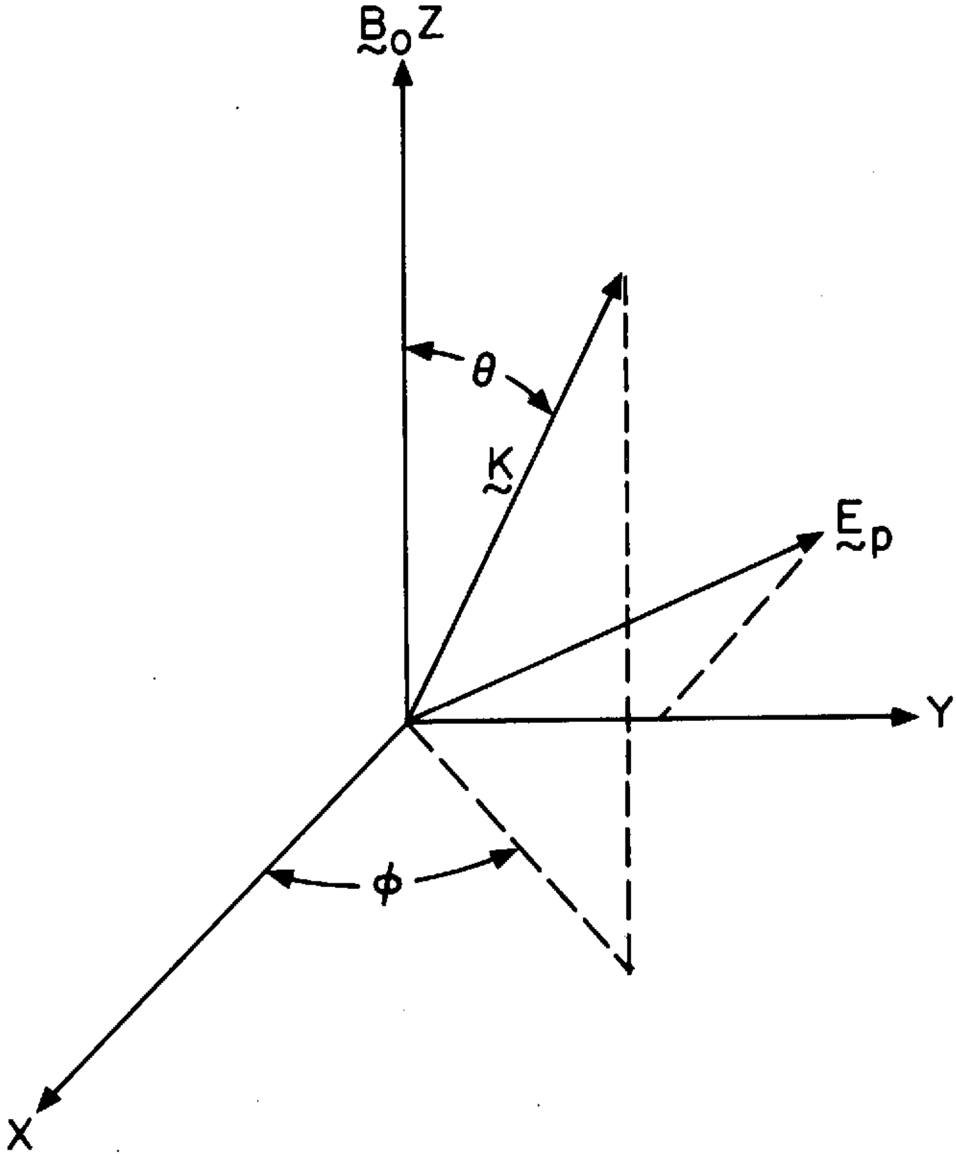
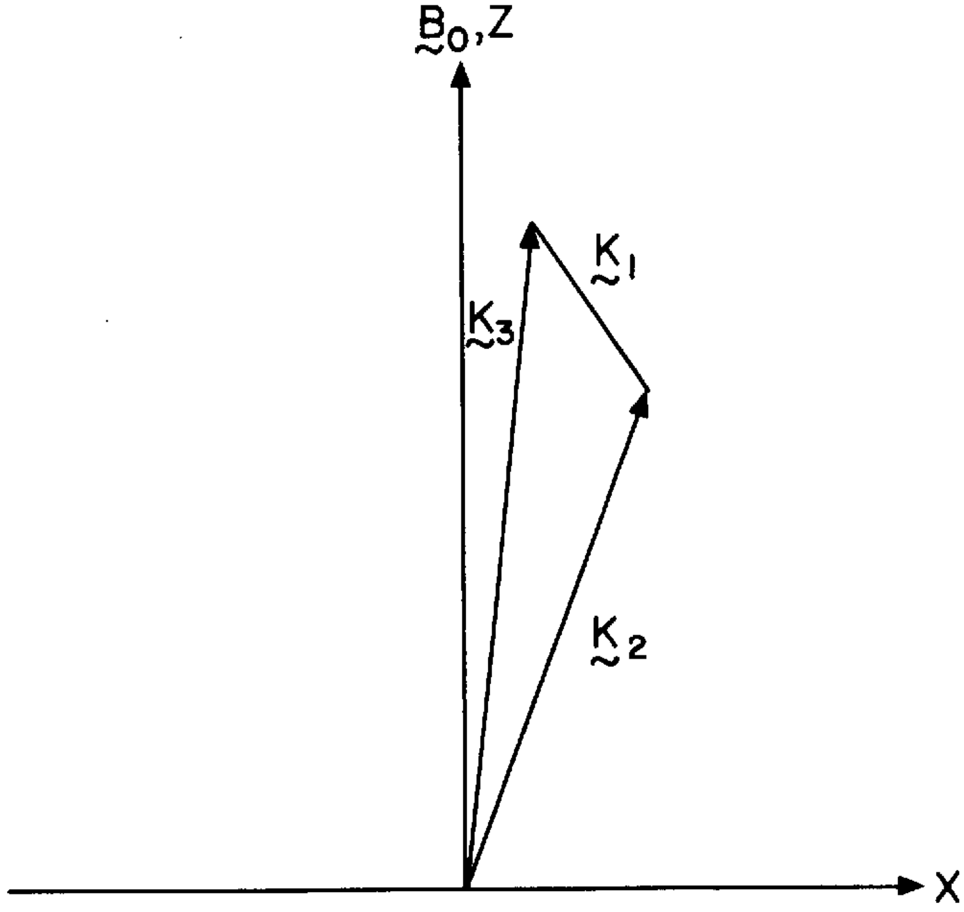


Figure 3



$\underset{\sim}{B}_0$,Z
$\underset{\sim}{K}_1$
$\underset{\sim}{K}_3$
$\underset{\sim}{K}_2$
X

Figure 4